Name of College - S.S. College, J. Bad
TOPIC — Differential Equation of First order and First Degree [Variable Separable] — class - B.Sc II Math (Hons) Group-B

Date -

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## Study Material → Variable separable

A differential Equation of the Form

$$M\,dx + N\,dy = 0$$

Where M, N are Functions of $x$ and $y$ ~~and~~ or are constants.

This is Differential Equation of First order and First degree.

The Equation Can be in the form

$$f(x)\,dx + g(y)\,dy = 0$$

Where $f(x)$ = Function of $x$ only
$g(y)$ = Function of $y$ only.

Then we say that variables are Separable

and its solution is

$$\int f(x)\,dx + \int g(y)\,dy = K$$

Where K is arbitrary constant.

Ex: → solve $\log\left(\frac{dy}{dx}\right) = ax + by$

Solution [ $\log_a x = N \Rightarrow e^N = x$ ]

Here $\log\left(\frac{dy}{dx}\right) = ax + by$

$$\Rightarrow \frac{dy}{dx} = e^{ax+by} = e^{ax} \cdot e^{by}$$

$$\Rightarrow \int e^{-by}\,dy = \int e^{ax}\,dx \Rightarrow \frac{e^{ax}}{a} - \frac{e^{-by}}{b} = K$$

$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + K$

which is the required solution

2. Solve $y\,dx - x\,dy = xy\,dx$

Solution $y\,dx - x\,dy = xy\,dx$

$$\Rightarrow x\,dy = y\,dx - xy\,dx$$

$$\Rightarrow x\,dy = y(1-x)\,dx$$

$$\Rightarrow \frac{dy}{y} = \frac{1-x}{x}\,dx$$

$$\Rightarrow \frac{dy}{y} = \left(\frac{1}{x} - 1\right)dx$$

On integrating, we get

$$\int \frac{dy}{y} = \int \left(\frac{1}{x} - 1\right)dx$$

$$\Rightarrow \log y = \log x - x + c$$

$$\Rightarrow \log y - \log x = c - x$$

$$\Rightarrow \log \frac{y}{x} = c - x$$

$$\Rightarrow \frac{y}{x} = e^{c-x}$$

$$\Rightarrow y = x e^{c-x}$$

which is the required solution

3. Solve $3e^{x} \tan y\,dx + (1-e^{x})\sec^2 y\,dy = 0$

Solution. Here Given Equation

$$3e^{x} \tan y\,dx + (1-e^{x})\sec^2 y\,dy = 0$$

$\Rightarrow (1-e^{x}) \sec^2 y \, dy = -3e^{x} \tan y \, dx$

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy = -\frac{3e^x}{1-e^x} dx$$

On integrating

$$\int \frac{\sec^2 y}{\tan y} dy = -3 \int \frac{e^x}{1-e^x} dx$$

$$\Rightarrow \log \tan y = 3 \log(1-e^x) + k$$

[ If $N^r$ is Diff. co-efficient of $D^r$ then integral of such is $\log D^r$ ]

which is the required solution.

Solve

4. $(1+y^2) dx + (1+x^2) dy = 0$

Solution :-> $(1+y^2) dx + (1+x^2) dy = 0$

$$\Rightarrow \frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$$

$$\Rightarrow \int \frac{dy}{1+y^2} = -\int \frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1} y = -\tan^{-1} x + c$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} y = c$$

$$\Rightarrow \tan^{-1} \frac{x+y}{1-xy} \tan^{-1} c$$

$$\Rightarrow \tan^{-1} \frac{x+y}{1-xy} = \tan^{-1} c$$

$\Rightarrow x+y = (1-xy) \tan c$ which is the required solution.

Ex.5 Solve.

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

Solution → Given Equation is

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$\Rightarrow \frac{dy}{dx} = e^x \cdot e^y + x^2 e^y$$

$$\Rightarrow \frac{dy}{dx} = [x^2 + e^x] e^y$$

$$\Rightarrow e^{-y} dy = (x^2 + e^x) dx$$

On ~~its~~ integrating

$$\int e^{-y} dy = \int [x^2 + e^x] dx$$

$$\frac{e^{-y}}{-1} = \frac{x^3}{3} + e^x + C$$

$$\Rightarrow e^x + \frac{x^3}{3} + e^{-y} = C$$ which is

the required solution.